Modelling and control of distillation column

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***Abstract*—Distillation column is one of the most important unit in a chemical plant. This paper provides a comprehensive study of mathematical model and control of binary distillation column. Wood-Berry distillation column model is considered in this paper which separates methanol from water. PID controller, decoupled PID controller and model predictive controller are used to control the distillation column.**

***Index Terms*—Distillation column; Wood-Berry Model; PID controller; Model Predictive Controller**

I. Introduction

Chemical process are multi variable and non-linear in nature. Due to relative uncertainties of the system, the chemical process are difficult to model as well as complex to control. Distillation is one of the most common unit operation used in most of the chemical and petroleum industries. Distillation is used to separate two or more components from a homogeneous fluid mixture. The quality and purity of distillation is of paramount importance, therefore proper control of the system is necessary.

A significant amount of literature is available on modeling and control of distillation column [**?**], [**?**]. This paper summarizes a few important literature. A tutorial perspective of dynamics and control of distillation column has been reported in [**?**]. Due to the nonlinear aspect of chemical process over a wide range of operating conditions, it is difficult to provide an accurate model of the system. The single linear time invariant (LTI) system is inadequate to describe the dynamics of the distillation column in both steady state as well as in transient state operation. In transient state when there is some major disturbances, the nonlinear characteristics of the process model becomes dominant. To counter this limitation of LTI model, linear parameter varying (LPV) model is used in process industries [**?**].

Researchers have tried to control distillation column using different conventional as well as intelligent control techniques. Terminal configuration control [**?**], model predictive control [**?**], [**?**], internal model control [**?**] and decentralized control [**?**] are some of the well known classical control algorithm used in distillation column.

Recently, intelligent control has been used in distillation column control which has generated a new research interest in this area. In intelligent control techniques, fractional PID and fractional order fuzzy PID controller has been used to control the distillation column in [**?**]. Control of distillation column using type-I and type-II fuzzy logic controller [**?**] and adaptive neuro-fuzzy (ANFIS) based controller [**?**] is reported in control literature. In [**?**], the researchers developed a self organizing fuzzy logic control (SOFLC) using general predictive control (GPC) on a Tagaki-Sugeno-Kang (TSK) model of controlled auto-regressive integrated moving average (CARIMA) structure. Neural network based control of distillation column is reported in [**?**].

This paper provides a comprehensive analysis of mathematical model and control of distillation column. Wood and Berry model of distillation column is considered which separates methanol from water. MIMO PID controller, decoupler based PID controller and model predictive controller are designed and performance of these controllers are evaluated using simulation platform.

The paper is organized as follows. Section II provides the basic working principle as well as mathematical modeling of distillation column. Section III provides the control strategy of the column and section IV provides results. Section V concludes the paper.

II. Distillation Column

A typical distillation column contains a vertical column where trays are used to enhance the component separations. A reboiler is used to provide heat for the necessary vaporization from the bottom of the column. Condenser is used to cool and condense the vapor from the top of the distillation column. Reflux drum is used to hold the condensed vapour so that liquid reflux can be recycled back from the top of the column. The distillation column contains one feed stream and two product streams. The feed contains a mole percent of the component called x_F. The product stream at the top has a composition referred as x_D. The product stream leaving the bottom contains a composition of x_B of the light component. The schematic diagram of distillation column is illustrated in Fig. **??**.

A. Mathematical Modelling

The mathematical model of the distillation column is provided below. Liquid holdup are assumed in every array of the distillation column. Francis-Weir formula is used for linearization and model the variable liquid hold up. It is represented as

$$L_n = L_{n0} + \frac{M_n - M_{n0}}{\beta} \tag{1}$$

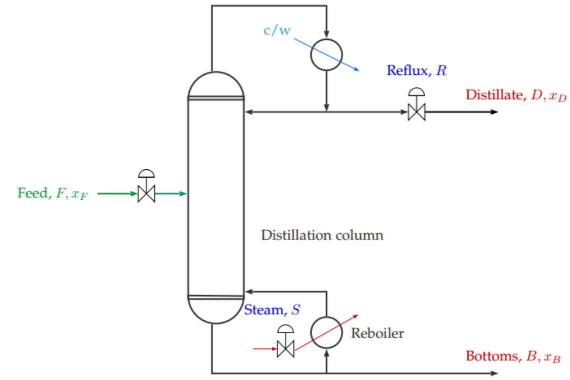

Fig. 1. Schematic diagram of distillation column

For condensor and reflux drum the mathematical model can be represented as

$$\frac{dM_D}{dt} = V_{NT} - (R + D_L + D_V) \tag{2}$$

$$\frac{dM_D X_D}{dt} = V_{NT}Y_{NT} - (R + D_L) X_D - D_V Y_D \tag{3}$$

For top tray the mathematical model can be represented as

$$\frac{dM_{NT}}{dt} = R + V_{NT-1} - L_{NT} - V_{NT} \tag{4}$$

$$\frac{dM_{NT}X_{NT}}{dt} = RX_D - L_{NT}X_{NT} + V_{NT-1}Y_{NT-1} - V_{NT}Y_{NT} \tag{5}$$

For nth tray the mathematical model is

$$\frac{dM_n}{dt} = L_{n+1} - L_n + V_{n-1} - V_n \tag{6}$$

$$\frac{dM_n X_n}{dt} = L_{n+1}X_{n+1} - L_n X_n + V_{n-1}Y_{n-1} - V_n Y_n \tag{7}$$

For feed tray the mathematical model is

$$\frac{dM_{NF}}{dt} = L_{NF+1} - L_{NF} + V_{NF-1} - V_{NF} + F_L \tag{8}$$

$$\frac{dM_{NF}X_{NF}}{dt} = L_{NF+1}X_{NF+1} - L_{NF}X_{NF} + V_{NF-1}Y_{NF-1} - V_{NF}Y_{NF} + F_L Z_L \tag{9}$$

For bottom tray the mathematical model is

$$\frac{dM_1}{dt} = L_2 - L_1 + V_B - V_1 \tag{10}$$

$$\frac{dM_1 X_1}{dt} = L_2 X_2 - L_1 X_1 + V_B Y_B - V_1 Y_1 \tag{11}$$

For reboiler the mathematical model is

$$\frac{dM_B}{dt} = L_1 - V_B - B \tag{12}$$

$$\frac{dM_B X_B}{dt} = L_1 X_1 - V_B Y_B - B X_B \tag{13}$$

B. Models of distillation column

Wood and Berry experimentally modelled a 9 inch diameter, 8-tray binary distillation column that separated methanol from water. The wood and berry model can be represented as

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.6e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + \begin{bmatrix} \frac{3.8e^{-8s}}{14.9s+1} \\ \frac{4.9e^{-3.4s}}{13.2s+1} \end{bmatrix} d \tag{14}$$

Here y_1 is distillate methanol $[mol\%]$, y_2 is water $[mol\%]$, u_1 is reflux flow rate $[lb/\min]$, u_2 is steam flow rate $[lb/\min]$, d is unmeasured flow rate $[lb/\min]$

III. Control Strategy

This section describes the control strategy for distillation column. The complete list of control variable and manipulated variable is shown in Fig. **??**. The schematic diagram of complete control structure of distillation column is shown in Fig. **??**.

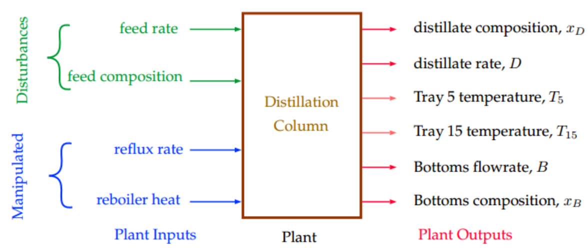

Fig. 2. Block diagram of control structure for distillation column

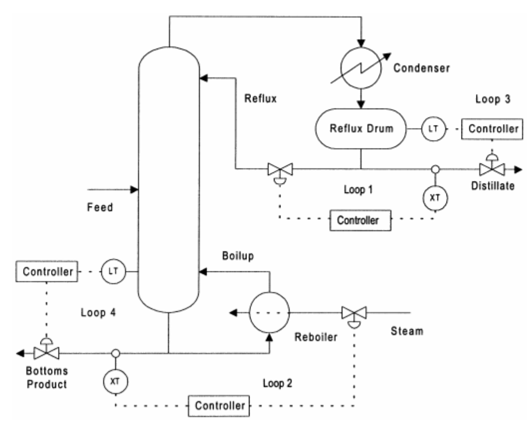

Fig. 3. Schematic diagram of control structure for distillation column

A distillation column has minimum 4 feedback control loops to control distillate concentration, bottom concentration, level of reboiler and level of reflux rate. So it is considered a MIMO control problem. This paper develops controller to control distillate concentration and bottom concentration. MIMO control can be controlled using two controllers, i.e each controller for one variable. The block diagram of PID

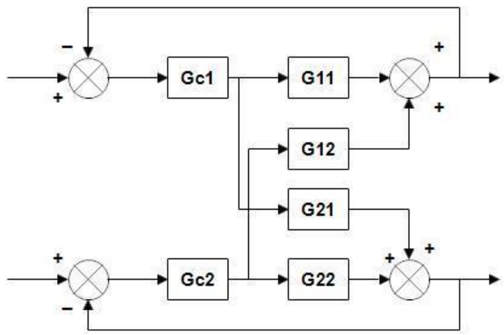

Fig. 4. PID control of MIMO plant

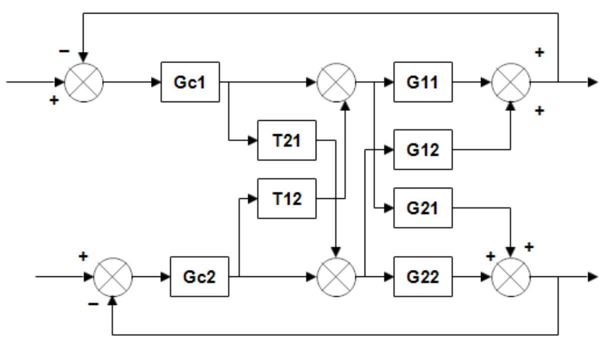

Fig. 5. PID control of MIMO plant with decoupler

controller based MIMO controller for distillation column is shown in Fig. **??**.

Due to multiple variables, the control schemes needs decoupler. The block diagram of control scheme with decoupler is shown in Fig. **??**. To check the interaction in a multi-loop control systems, decouplers are used. Static decouplers are used when fast controls are not required. They possess simple simple structure and their design does not require detailed description of the system. Relative Gain Array is a normalized form of the gain matrix that describes the impact of each control variable on the output, relative to each control variable impact on other variables. The decoupler can be found out using following equations $T_{12} = \frac{-G_{12}}{G_{11}}$ and $T_{21} = \frac{-G_{21}}{G_{22}}$

A. Model Predictive Control

In a MIMO control, the output vector can be represented as $y = \begin{bmatrix} y_1 & y_2 & -- & y_m \end{bmatrix}^T$ and input vector can be represented as $u = \begin{bmatrix} u_1 & u_2 & -- & u_r \end{bmatrix}^T$

The MIMO model for the corrected prediction can be represented as

$$\tilde{Y}(k+1) = S\Delta U(k) + \hat{Y}^o(k+1) + \phi[y(k) - \hat{y}(k)] \quad (15)$$

$$\tilde{Y}(k+1) = \begin{bmatrix} \tilde{y}(k+1) \\ \tilde{y}(k+1) \\ : \\ \tilde{y}(k+P) \end{bmatrix}$$

$$\hat{Y}^o(k+1) = \begin{bmatrix} \hat{y}^o(k+1) \\ \hat{y}^o(k+1) \\ : \\ \hat{y}^o(k+P) \end{bmatrix}$$

$$\Delta U(k) = \begin{bmatrix} \Delta u(k) \\ \Delta u(k+1) \\ : \\ \Delta u(k+M-1) \end{bmatrix}$$

The dynamic matrix is defined as

$$S = \begin{bmatrix} S_1 & 0 & .. & 0 \\ S_2 & S_1 & 0 & : \\ : & : & & 0 \\ S_M & S_{M-1} & .. & S_1 \\ S_{M+1} & S_M & .. & S_2 \\ : & : & & : \\ S_P & S_{P-1} & .. & S_{P-M+1} \end{bmatrix}$$

Fig. **??** shows the block diagram of model predictive control for distillation column.

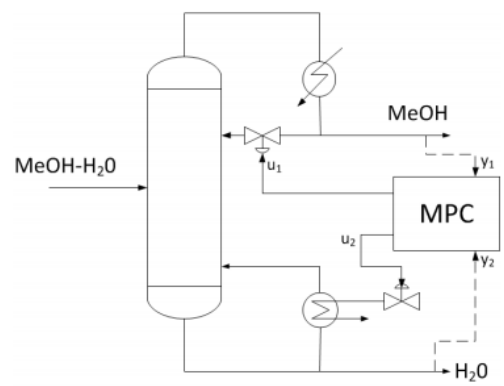

Fig. 6. Block diagram of control structure for distillation column

The quadratic performance index used for model predictive controller is expressed as

$$\min_{\Delta U(k)} J = \hat{E}(k+1)^T Q \hat{E}(k+1) + \Delta U(k)^T R \Delta U(k) \quad (16)$$

where Q is a positive-definite weighting matrix and R is a positive semi-definite matrix. Both are diagonal matrices with positive diagonal elements.

The MPC control law is derived by minimizing the quadratic objective function. The MPC control law can be written as

$$\Delta U(k) = K_c \ \hat{E}^o(k+1) \quad (17)$$

Here $K_C = (S^T Q\, S + R)^{-1}\ S^T Q$

IV. Simulation Results

This section provides simulation results for the distillation column. Wood and Berry model is considered as the model for the distillation column. First of all simulation for open loop step response analysis and open loop transient analysis is carried out. The open loop step response of binary distillation column is shown in Fig. **??**.

Open loop transient analysis is studied for the distillation column. Once the flow rate is varied, the response of output variable is studied in Fig. **??** and Fig. **??** respectively.

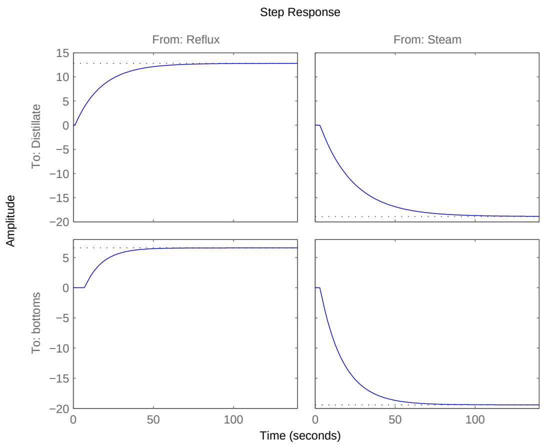

Fig. 7. Open loop step response analysis

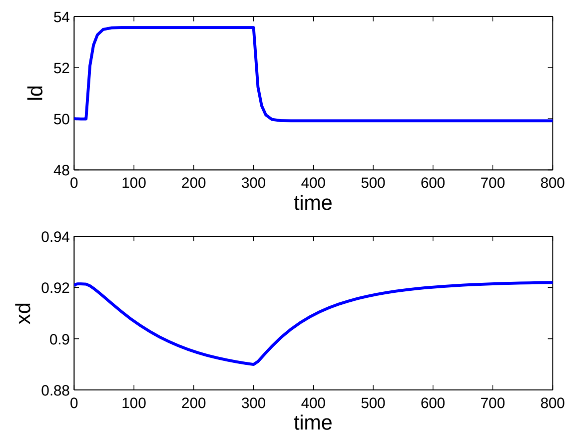

Fig. 8. Change in flow rate of distillation column

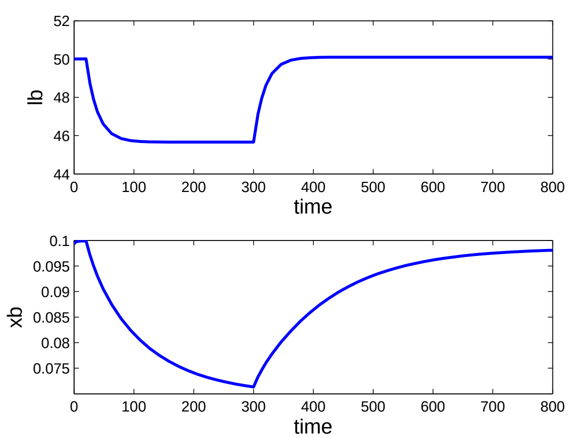

Fig. 9. Change in flow rate of distillation column

PID controller without decoupler is used to control the MIMO plant. Wood and Berry distillation column model is considered and PID controller is tuned using Zigler-Nichols tuning method. The controlled output of the system with PID controller is shown in Fig. **??**. As it can be seen from the graph, the distillate and the bottom product are regulated using PID controller but there is significant overshoot in the control scheme.

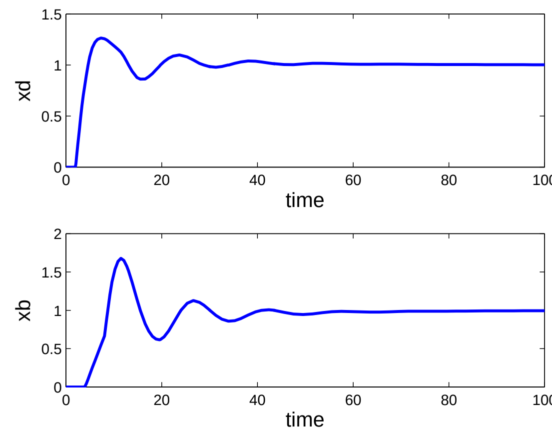

Fig. 10. Response of PID controller without decoupler

Fig. **??** presents the output response of model predictive controller in a distillation column. It can be shown in the graph that the model predictive control algorithm provides the best output response for a distillation column.

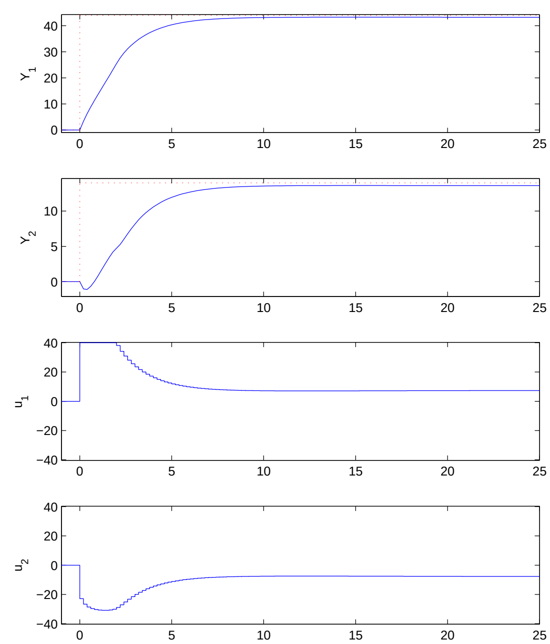

Fig. 11. Response of model predictive controller

V. Conclusion

This paper provides a detailed mathematical model of a distillation column and control part of the distillation column has been studied extensively. For simulation purpose, Wood-Berry distillation column model is considered. PID controller for MIMO plant is designed without decoupler and with decoupler unit. Model predictive controller is also studied and implemented for a distillation column. The performance of the control strategy is evaluated with the help of simulation analysis.

Nomenclature

Symbol	Unit	Description
L_n		Internal liquid flow rate
L_{n0}		Reference value of internal flow rate
M_{n0}		Reference molar holdup for nth tray
M_D	lbmol	Liquid holdup in reflux drum
R	lbmol/h	Reflux flow rate
D_L	lbmol/h	Flow rate of liquid distillate
D_V	lbmol/h	Flow rate of vapour distillate
X_D	mol fraction	Composition of liquid distillate
Y_D	mol fraction	Composition of vapour distillate

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